Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2013

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section One: Calculator-free

Please place your student identification label in this box

Student Number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 13-095
MAS3CD-S1

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	11	11	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

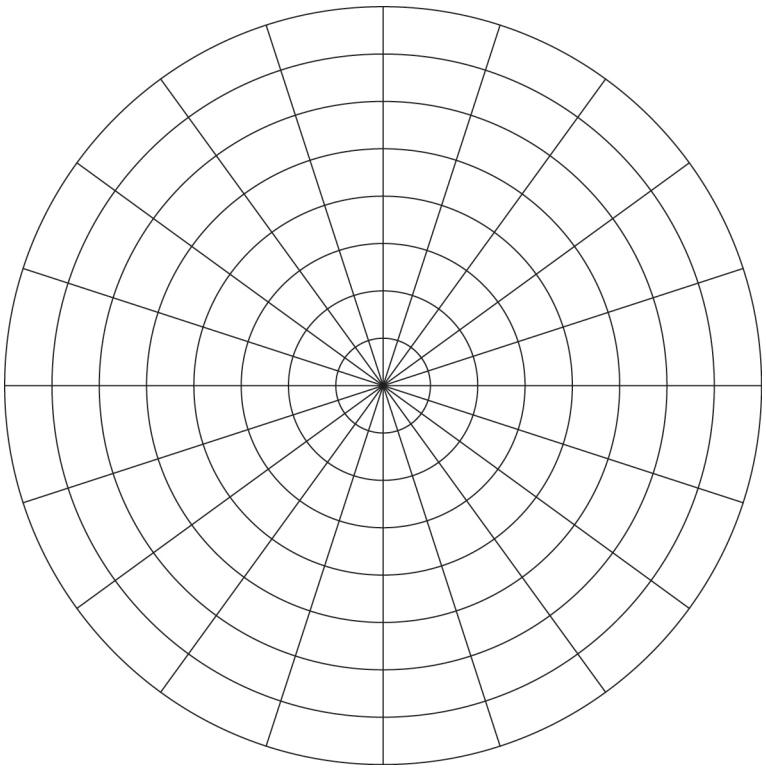
Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(5 marks)**

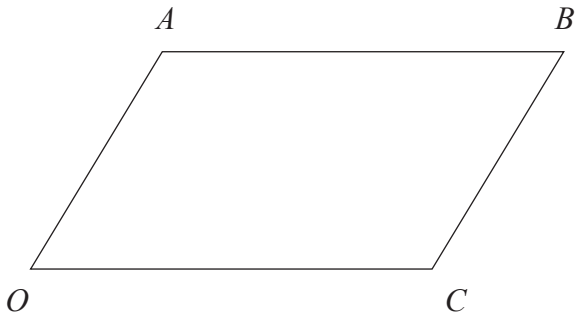
(a) Plot the roots of $z^5 = 1$ in the complex plane on the diagram below, given that $z = 1$ is one of the roots. (3 marks)



(b) The roots above form the vertices of a pentagon. Determine the exact value for the perimeter. (2 marks)

Question 2 **(6 marks)**

Let $OABC$ be a parallelogram with sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ denoted by vectors **a** and **c** respectively.



(a) Express the diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of **a** and **c.** (1 mark)

(b) Prove the parallelogram law that states:
'The sum of the squares of the lengths of the diagonals of a parallelogram equals the sum of the squares of the lengths of all four sides.' (5 marks)

Question 3 **(9 marks)**

Determine the following integrals:

(a) $\int \frac{t-4}{\sqrt{t+3}} dt.$ (3 marks)

(b) $\int 5x^2 e^{-7x^3} dx.$ (3 marks)

(c) $\int \sin^2(3x) \cos^2(3x)\, dx.$ (3 marks)

See next page

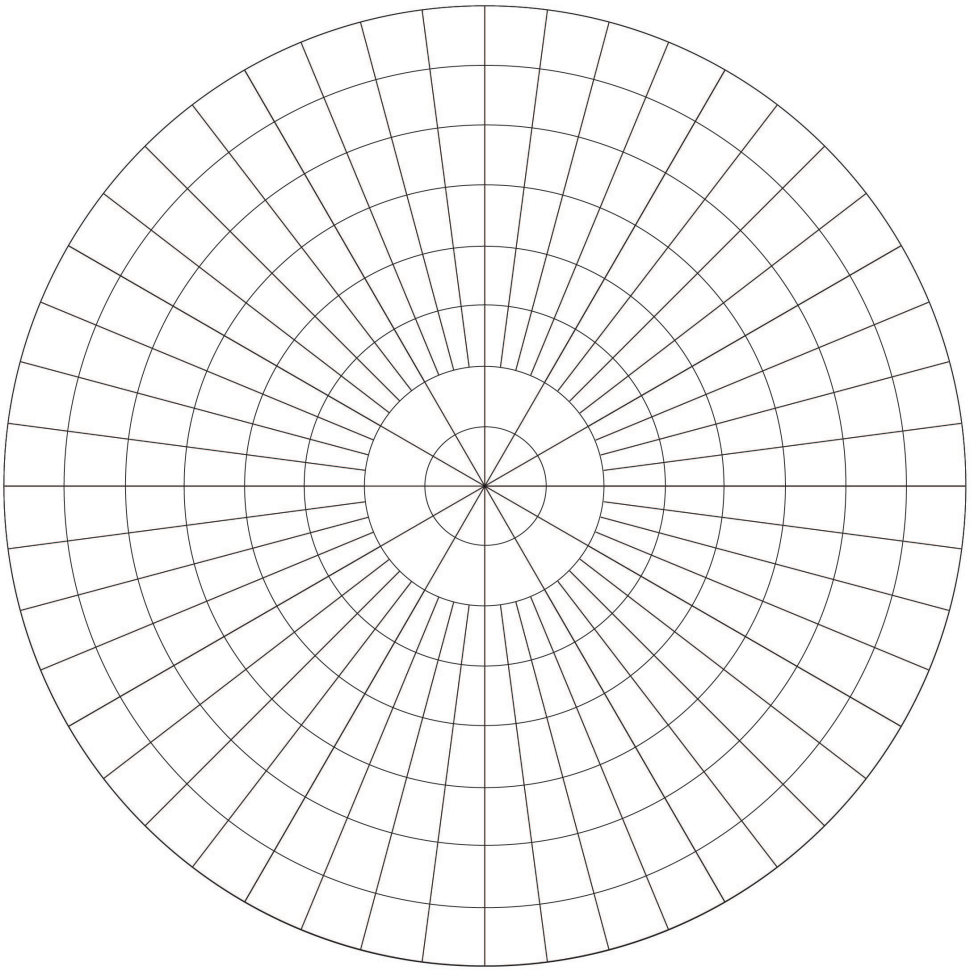
Question 4 **(8 marks)**

(a) Sketch the following two polar graphs on the axes below:

(i) $r = 4$.

(ii) $\theta = \frac{\pi}{3}$.

(iii) Let point A be the point of intersection of these two graphs. Mark this point on your sketch. (4 marks)



See next page

(b) Sketch the polar graph $r = \theta$ on the graphs shown in part (a). Label the point B on this curve where $\theta = \frac{11\pi}{6}$. (2 marks)

(c) State an expression for the exact distance between points A and B. (2 marks)

See next page

Question 5 **(7 marks)**

Consider the 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$.

(a) Prove that $\det(A)\det(B) = \det(AB)$. (3 marks)

(b) Prove by the method of mathematical induction that $\det(A^n) = (\det A)^n$ for $n = 1, 2, 3...$
(4 marks)

See next page

Question 6 **(4 marks)**

Use logarithmic differentiation to find $\frac{dy}{dx}$ where $y = (1 - 3x)^{\cos x}$.

See next page

Question 7 **(4 marks)**

Determine the complex number $z = a + bi$, where a, b are real constants with $a > 0$ such that $\text{Im}\left(\frac{1}{z^2}\right) = \frac{1}{100}$ and $\text{Im}(z) = -2\text{Re}(z)$.

See next page

Question 8 **(7 marks)**

The natural logarithm function $\ln(x)$ is defined by $\ln(x) = \int_1^x \frac{dt}{t}$, which is valid for $x > 0$.

Use this definition (making no assumptions about the logarithm laws) to prove, from first principles, that for all $x, y > 0$.

(a) $\ln(1) = 0$. (1 mark)

(b) $\ln\left(\frac{1}{x}\right) = -\ln(x)$. (Hint: make the substitution $u = \frac{1}{t}$ in the definition of $\ln\left(\frac{1}{x}\right)$.)

(3 marks)

(c) $\ln(xy) = \ln(x) + \ln(y)$. (3 marks)

End of questions

Additional working space

Question number: _____

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Question number: _____

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Published by the School Curriculum and Standards Authority of Western Australia
27 Walters Drive
OSBORNE PARK WA 6017